Ahmed Elgamal (Draft, 2010)

Bracing for Earthquake Resistant Design

September 18, 2002 (2010 update)

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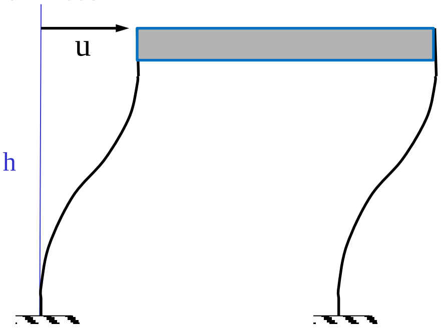
Rigid Roof Idealization and Column Stiffness



Relative to the columns, the roof structural system might be quite rigid, resulting in the classical deformed shape shown in the Figure.

In this deformed configuration, the column stiffness is dictated by a state of fixed-fixed boundary conditions.

In order to define the lateral column stiffness (k) for this fixed-fixed state, we start with the beam Equation of equilibrium and the appropriate boundary conditions (see next page for derivations):

$EI\, w'''' = 0$

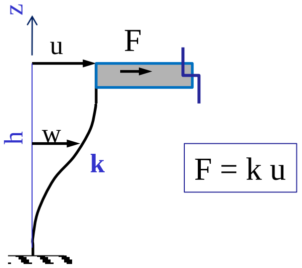
$w(0) = w'(0) = w'(h) = 0$, and $w(h) = u$

Resulting in: $w = \left((3z^2/h^2) - (2z^3/h^3) \right) u$

With shear force $Q = - EI\, w'''$, the shear force F at h becomes:

$F = -EI w'''(h) = (12EI / h^3)\, u$

and $k_{(column)}$ is therefore, $k = (12EI / h^3)$



Bending beam Equation

$EI\,w'''' = 0$ (case of zero pressure acting along the beam length)
$EI\,w''' = c_1$
$EI\,w'' = c_1 z + c_2$
$EI\,w' = c_1 z^2/2 + c_2 z + c_3$ (slope)
$EI\,w = c_1 z^3/6 + c_2 z^2/2 + c_3 z + c_4$ (displacement)
$w(0) = 0$ results in $c_4 = 0$
$w'(0) = 0$ results in $c_3 = 0$
$w'(L) = 0$ results in $c_2 = -(c_1/h)$
$w(L) = u$ results in $c_1 = -(12\,EI/h^3)\,u$
Therefore $w = ((3z^2/h^2) - (2z^3/h^3))\,u$

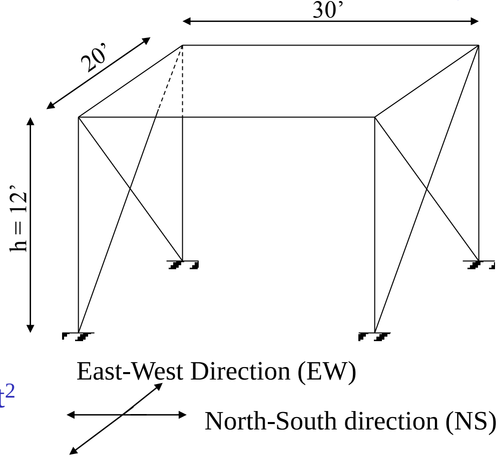
Note:
Moment (M) = $-EI\,w''$
Shear force = $M' = -EI\,w'''$

Bracing

Building is supported laterally by:

1) Bending stiffness of 4 I-beams in the NS and EW directions

2) Axial stiffness of 4 slender rod braces in the EW direction



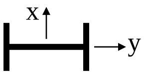
Mass
Take weight (w)of roof as 30 lb/ft^2 and calculate the mass (m)

$$m = \frac{w}{g} = \frac{30\times30\times20}{386.4} = 46.63\,\text{lb-sec}^2/\text{in} = 0.04663\ \text{kip-sec}^2/\text{in}$$

Note: Acceleration of gravity (g) = 386.4 in/sec^2

E_s = 29,000 ksi (Steel Young's Modulus)

Steel I-Beam columns (Section a-a)



(W8x24 Steel I-beam)

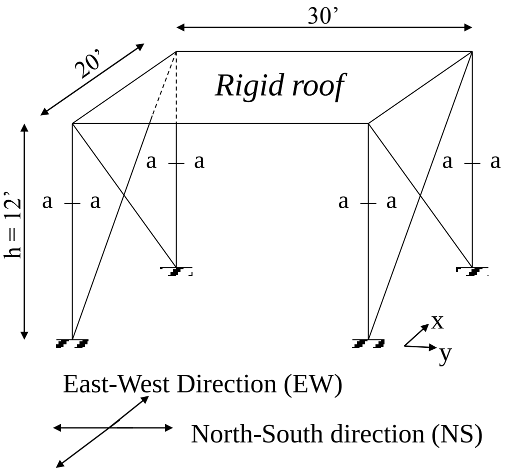
$I_x = 82.8\ \text{in}^4$

$I_y = 18.3\ \text{in}^4$

Note that I_y is much smaller than I_x

Brace (1 in diameter circular bar)

Cross sectional area $A = 0.785\ \text{in}^2$



In NS-direction

$$k_{NS} = 4\left(\frac{12EI_x}{h^3}\right) = 4\frac{12\left(29x10^3\right)(82.8)}{(12x12)^3} = 38.58\ \text{kips}/\text{in}$$

NS-direction Equation of motion:

$$m\left(\ddot{u}_{NS} + \ddot{u}_{NS_g}\right) + c\dot{u}_{NS} + k_{NS}u_{NS} = 0$$

where $c = \zeta c_{cr} = \zeta\left(2\sqrt{km}\right)$ and ζ Maybe ≈ 1.2-1.5% for steel

or $\left(\ddot{u}_{NS} + \ddot{u}_{NS_g}\right) + 2\zeta_{NS}\,\omega_{NS}\,\dot{u}_{NS} + \omega_{NS}\omega_{NS}u_{NS} = 0$

where $\omega_{n(NS)}$= sqrt (38.58 / 0.04663) = 28.76 radians/sec

Note: $f_{n(NS)}$ = 28.76 / (2 x 3.1428) = 4.58 Hz (cycles/sec)

and $T_{n(NS)}$ = 1/ 4.58 = 0.22 seconds

In EW-direction

$$k_{EW} = 4\left(\frac{12EI_y}{h^3}\right) = 4\frac{12(29x10^3)(18.3)}{(12x12)^3} = 8.52 \quad kips/in$$

As can be seen, lateral stiffness of the columns in the EW direction is much lower than that in the NS direction. The braces will change this situation dramatically.

Thus, we will rely on brace stiffness since column stiffness is relatively small and not intended for lateral support (only to carry vertical load).

Laterally, $f_s = k_{brace}\, u$, or $k_{brace} = f_s / u$

From geometry, $f_s = p \cos\theta$ and $u = (\delta / \cos\theta)$

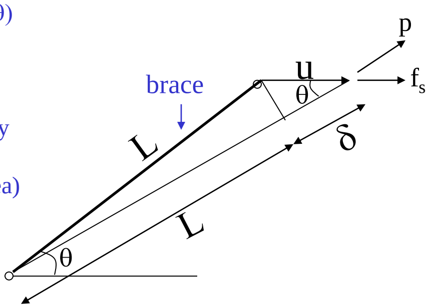
resulting in $k_{brace} = (p/\delta)\cos^2\theta$

In the brace, axial stress is related to axial strain by

$(p/A) = E_s(\delta/L)$ (A is brace cross-sectional area)

so that $(p/\delta) = (AE_s/L)$, and therefore

$$\mathbf{k_{brace} = (AE_s / L) \cos^2 \theta}$$



From the building geometry

$$\cos\theta = \frac{20}{\sqrt{12^2 + 20^2}} = 0.8575$$

L = sqrt (20² + 12²) = 23.3 ft

and $$k_{brace} = \frac{0.785(29\times10^3)}{23.3\times12}(0.8575)^2 = 59.8 \text{ kips/in}$$

Braces are slender in this case, and therefore only provide added stiffness when subjected to tensile force (the braces sag or buckle when in compression. As such, only two braces will be providing lateral stiffness at any given time. As such,

$$k_{EW\,(bracing)} = 2\times59.8 = 119.6 \text{ kips/in}$$

Since the stiffness due to bracing is much larger than that due to the 4 columns, we will only rely on the bracing for stiffness in this direction:

$$m(\ddot{u}_{EW} + \ddot{u}_{EW_g}) + c\dot{u}_{EW} + k_{EW}\,u_{EW} = 0 \quad \text{or,} \quad (\ddot{u}_{EW} + \ddot{u}_{EW_g}) + 2\zeta_{EW}\,\omega_{EW}\,\dot{u}_{EW} + \omega_{EW}\omega_{EW}\,u_{EW} = 0$$

where $\omega_{n(EW)}$= sqrt (119.6 / 0.04663) = 50.64 radians/sec

Note: $f_{n(EW)}$ = 50.64 / (2 x 3.1428) = 8.06 Hz (cycles/sec)

and $T_{n(EW)}$ = 1/ 8.06 = 0.12 seconds

Question: *Will all 4 braces in the frame be effective at the same time?*

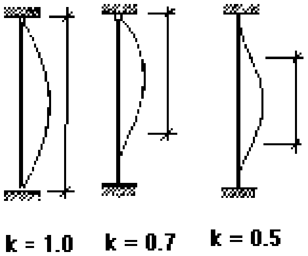
Euler Buckling Load http://en.wikipedia.org/wiki/Buckling :

$$Ncr = \frac{\pi^2 EI}{L_k^2}$$, $\mathbf{L_k}$ = k L = Effective Buckling Length

$$I = \frac{\pi r^4}{4} \qquad I = \frac{\pi (0.5")^4}{4} = 0.049\,\text{in}^4$$

$$L_k = \frac{1}{1}(23.3\text{ft}) = 23.3\text{ft} = 279.6\,\text{in}$$

$$N_{cr} = \frac{\pi^2 (29000\text{ksi})(0.049\text{in}^4)}{(279.6\text{in})(279.6\text{in})} = 0.179\,\text{kips} = 179\text{lbs}$$



Answer: When the brace experiences compression, buckling load is minimal. This is why, we used only two braces at a time in our calculations (of the 4 installed braces)

Buckling Restrained Braces (BRBs)

http://jiano.typepad.com/photos/uncategorized/brb_02.jpg

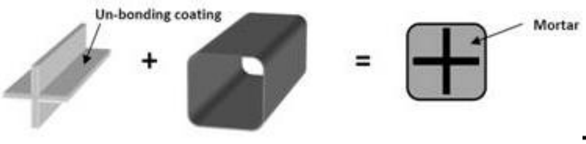


Figure 1 – Typical Buckling-restrained brace configuration. A steel core plus a steel restraining tube and mortar, make up the BRB cross section.

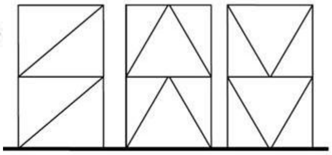
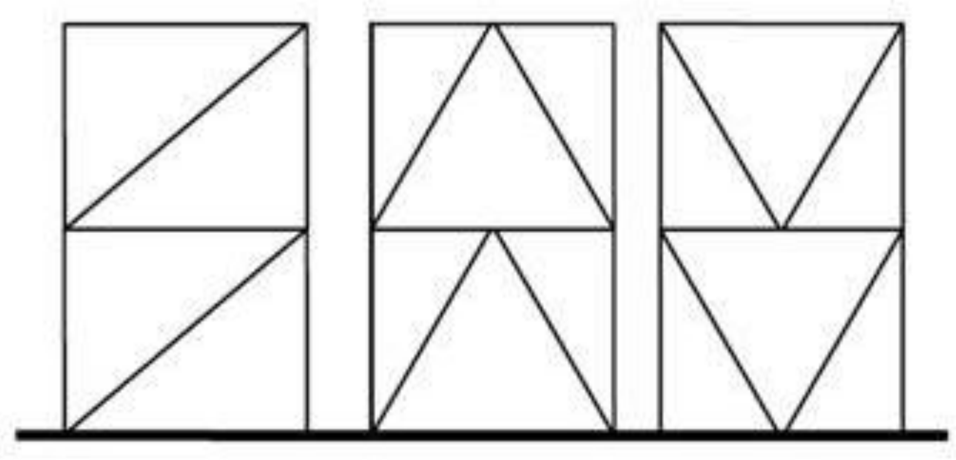
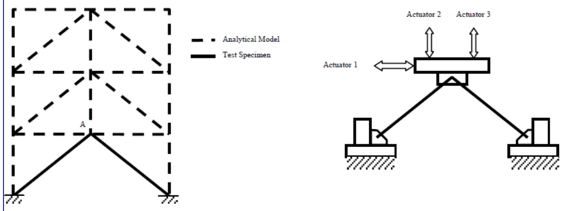


Figure 12 – Three types of CBF configurations. From left to right (a) Diagonal (b) Chevron (c) V-Braced. [20]



Bracing Systems (a) Diagonal, b) Chevron, and c) V-Braced



(a) Analytical and Experimental Substructures

(b) Test Setup

Figure 6 – Fast Hybrid Tests at the University of Colorado

Behavior of Braced Steel Frames with Innovative Bracing Schemes

A NEES Collaboratory Project

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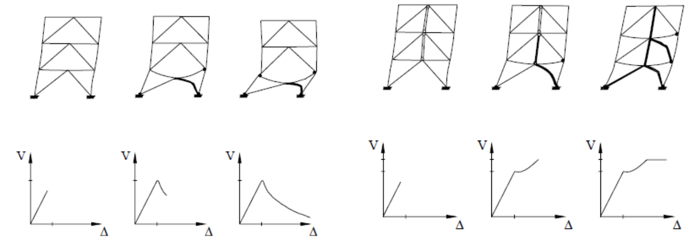

(a) Conventional braced frame

(b) Zipper frame

Figure 2 – Comparison of the collapse mechanism and load-displacement relationships for zipper and conventional braced frames (Khatib et al. 1988).

Testing

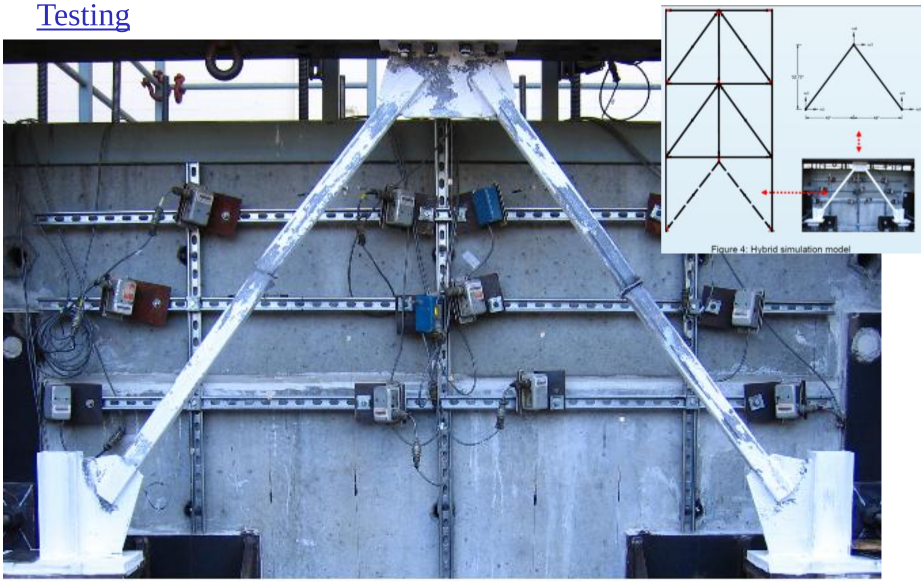
Figure 4: Hybrid simulation model

http://peer.berkeley.edu/~yang/NEESZipper/Summary.html


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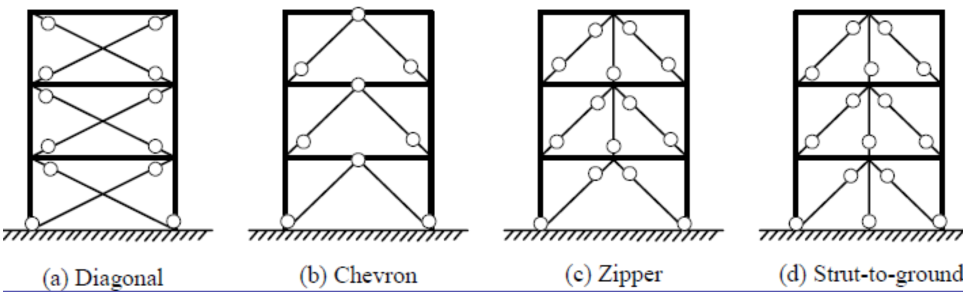

(a) Diagonal
(b) Chevron
(c) Zipper
(d) Strut-to-ground

Figure 1 – Typical braced frame configurations (circles denote possible locations for energy dissipators).

http://nees.buffalo.edu/projects/zipperframes/CollaboratoryResearchGT-UCB-UCB-UB-USF.pdf

Energy Dissipation Devices

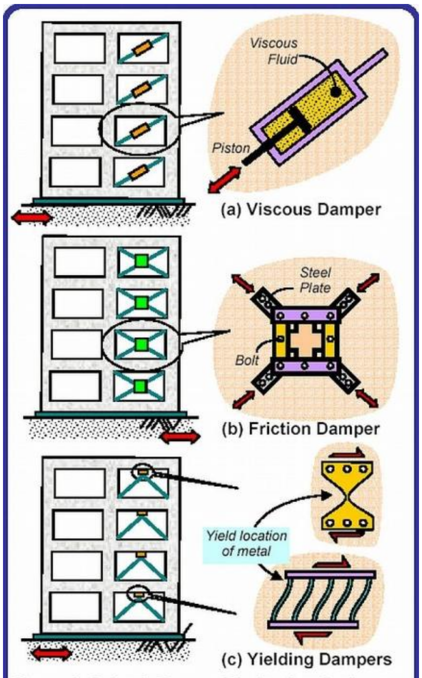
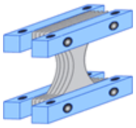
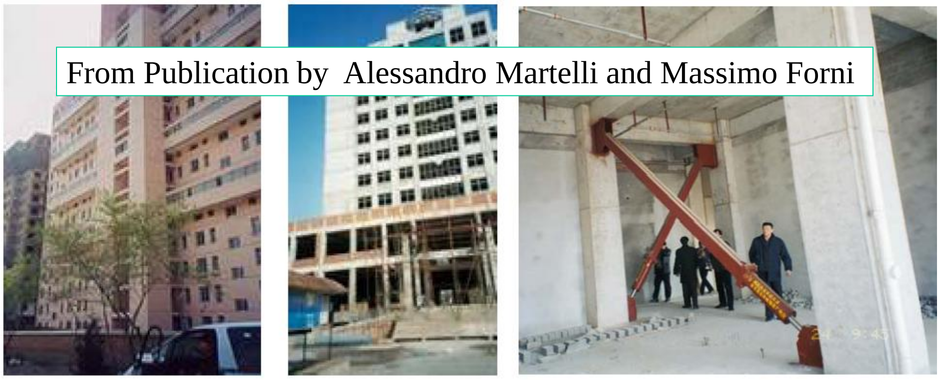


Figure 3: Seismic Energy Dissipation Devices – *each device is suitable for a certain building.*



Viscous Dampers



From Publication by Alessandro Martelli and Massimo Forni

Figure 12. The tallest Chinese seismically isolated building (19 storeys), erected at Taiyuan City, in Northern China (left), and a Chinese high-rise building protected by VDs (center and right).