

## Homework

Experimental verification of the linear vibration of a 2 DOF system based on Webshaker site <http://webshaker.ucsd.edu>. Note that the installed models exhibit extremely low damping (e.g.  $\zeta = 0.001$  or  $0.0015$ , i.e.,  $\zeta = 0.1\%$  or  $0.15\%$ ). Response with higher damping can be assessed based on a calibrated numerical model in which higher damping ratios can be specified by the user. Also, the masses are the same for both floors.

1) Go to 2DOF model and choose sinusoidal input. Using the default values (normalized amplitude = 0.2, input frequency = 4 Hz, and shaking time = 10 sec.):

- a) What was the peak relative displacements (floor 1 and floor 2)?
- b) Go to frequency domain (floor displacements) and find accurate values for the first two natural frequencies Hz.

2) Repeat above, but go ahead and shake the system at its first resonance and record maximum relative displacement.

Why is the displacement amplitude increasing in every cycle of excitation in this case?

During this forced vibration response at resonance, the model is vibrating at the first resonance. Therefore, it is exhibiting the first mode shape response with an amplitude varying with time (oscillating back and forth). Go ahead and define this mode shape by say going to a peak displacement, and recording the relative displacement of floors 1 and 2 (this defines  $\phi_{11}$  and  $\phi_{21}$  for mode  $\phi_1$ ). Draw a sketch of this first mode. As you know, the mode is fully defined by the ratio between  $\phi_{11}$  and  $\phi_{21}$ . This means that at any other instant of time, this ratio should be maintained. Therefore, go ahead and choose another time instant (does not have to be at peak displacement), and measure  $\phi_{11}$  and  $\phi_{21}$ . You should get the same ratio between  $\phi_{11}$  and  $\phi_{21}$ , showing that the mode shape is preserved during vibration.

Note that the two floors are moving in phase (i.e., they oscillate reaching peak displacement or zero displacement at the same time instants). This is a characteristic of systems with proportional damping characteristics (in the model, damping is very very low, helping this nearly perfect in-phase response).

3) Repeat above, but go ahead and shake the system at its second resonance. You will see that  $\phi_{12}$  and  $\phi_{22}$  are out of phase (i.e., peak positive of floor 1 corresponds to peak negative of floor 2). Please keep in mind that superfluous measurement noise, and shaking using a frequency that is not **exactly** the resonant frequency might affect the results minimally (you should still see a very clear second mode).

4) Now you have the resonant frequencies and the mode shapes. Damping is very very small (of the order of 0.0015 or damping ratio of 0.15 %) in both modes. Please write down the steps involved in obtaining a modal solution for a base earthquake excitation. Note that you don't need to know the stiffnesses or masses of the structure, now that you know the resonant frequencies and mode shapes.

How can you use this modal solution to predict what a similar system with say 5 % viscous damping in both modes would respond?

5) Go to Webshaker vs. 2DOF (earthquake like-input), plug in your mode shapes, resonant frequencies, and damping ratios, and shake using the Imperial Valley 1940 record. How good is your numerical prediction?

Run a more realistic numerical simulation with 2 % viscous damping in each mode, and comment on the result (include a plot of the response).

6) Very important:

For future improvements of this homework effort, please comment on the Webshaker homeworks parts 1 and 2. Any advise for us are most appreciated. Many thanks in advance.