Assignment
Single-Degree-of-Freedom (SDOF) and Response Spectrum

Question No. 1

Assume the elevated steel tank shown in Figure 1 can be modeled as a single-degree-of-freedom (SDOF) system as shown in Figure 2.

A) Given the weight of tower (W) is 800 tons (English Units) and the resonant frequency is 3 Hz, determine the equivalent stiffness (k) of the SDOF system, angular frequency $\omega$, and period T.

B) Repeat part A using SI units with:
Weight of tower = 7000 kN
Natural Frequency = 3 Hz

C) For the structure in part A:
If this structure is subjected to the El Centro earthquake, find the peak relative displacement from the El Centro S00°E response spectrum.
Repeat the above for the 2% and 10 % damping ratios situations.

Note: Please refer to “Single-Degree-of-Freedom (SDOF) and Response Spectrum” notes for the El Centro response spectrum (or see Figure included below).
Question No. 2

a) Convert $ma + cv + ku = - ma_{\text{ground}}$ to an equation in terms of frequency and damping ratio instead of $m$, $c$ and $k$.

b) Define the damping ratio in terms of $c$ and $c_{\text{crit}}$

c) How does a system respond if damped above $c_{\text{crit}}$ (Draw a sketch to illustrate please)

d) What can we learn from the free vibration phase of a SDOF system (include a time history diagram)?

e) For a SDOF of $T = 2$ sec and 2% viscous damping, $D = 8$ inches (in a particular spectrum). Write expressions for $V$ and $A$ for this SDOF (include units, and change $A$ to $g$ units please).

f) How do we get frequency in Hz using $k$ and $m$.

g) Given the weight ($W$) of a SDOF structure is 7000 kN and its stiffness ($k$) is 250 MN/m, find the peak relative displacement, pseudo velocity, pseudo acceleration, and peak ground acceleration (PGA) from the El Centro S00°E response spectrum below for the 2% and for the 10% damping cases.

h) In a SDOF idealization, $m = 2$ kg, find $k$ (mention units) for a natural period of 2.5 seconds.

i) Draw a labeled sketch of a SDOF idealized structure

j) What is a typical damping ratio for a Structure in (%)

k) What’s a typical damping ratio for a Structure, as a number in the SDOF equation

l) Define Hz (draw a small sketch if needed).

m) A viscous damping $c$ is added in the SDOF equation, although we know that viscosity is not necessarily the dominant damping mechanism in structures. Mention some common damping mechanisms in structures. Why do we resort to $c$ in our analyses?
Question No. 3: (About Elastic Design Spectra)

For the following site information:

Maximum Ground Acceleration = 0.35 g
Maximum Ground Velocity = 16 in / s
Maximum Ground Displacement = 12 in

Using a damping ratio of 2%, construct the Elastic Design Spectra for mean and mean +1 standard deviation (please use the attached Figure below).

Note: see section 6.2 "Elastic Design Spectrum" from Chopra's Dynamics of Structures Book for appropriate amplification factors (also included in class notes).
Figure A6.1: Graph paper with four-way logarithmic scales.

Natural vibration period $T$, sec.

Pseudo-velocity $V$, in/sec.

Pseudo-acceleration $A_g$.