Seismic Waves

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Types of Seismic Waves

Body waves

1. Primary or P-wave (Compression Wave)
   Pushes (compresses) or Pulls (dilates)

   Propagates through solids and fluids.
   May emerge to ground surface as audible sound wave
   (to animals and humans, if >15 cycles per second).
Let: $B =$ Modulus of incompressibility / bulk modulus  
$G =$ Modulus of rigidity / shear modulus (G or $\mu$)  
$E =$ Young’s modulus  
$\nu =$ Poisson’s ratio  
$\rho =$ Density  
$B$ and $G$ can be expressed in terms of $E$ and $\nu$  

P-wave velocity $\alpha = \sqrt{(B + \frac{4}{3}G)/\rho} = V_p$  

For granite $\alpha = 4.8 \text{ km/sec}$  
For water $\alpha = 1.4 \text{ km/sec}$
2. Secondary or S-wave (Shear Wave)
Shears the rock at right angles to its travel path

Travels through solids only
S-wave velocity \( \beta = \sqrt{G/\rho} = V_s \)
For granite, \( \beta = 3.0 \text{ km/sec} \)
For water, \( \beta = 0 \text{ km/sec} \) (\( G \) or \( \mu \) of fluids = 0)

P-wave is faster than S-wave
Arrives first to a site during an earthquake. S-wave follows after a few seconds and causes most damage due to side-to-side motion. Most earthquake energy is in shear with high shaking amplitude.
**Surface waves**
These are waves with motion restricted to near ground surface (wave displacement decreases with depth, like ripples of water across a lake).

**Love waves**
Similar to S-wave with amplitude decreasing with depth (only horizontal motion)

Love waves do not propagate in fluids.

**Rayleigh waves**
Causes vertical and horizontal motion (elliptical movement) in a vertical plane along propagation path.
Surface waves travel slower than body waves. 
(Love waves generally travel faster than Rayleigh waves)

Velocity: P-wave > S-wave > Love wave > Rayleigh wave (in general)

Wave Reflection and Refraction

Example

Incident from source of earthquake

Reflected

Refracted

Rock-rock (different type) or soil-rock boundary

Softer rock

Stiffer rock
Vertical wave incidence near ground surface

Soil Stiffness

1-Dimensional Shear Beam Site Amplification

Wave Refraction

Earthquake Source
Soil Stiffness

After: http://web.ics.purdue.edu/~braile/edumod/slinky/slinky.htm
Wave Propagation

- P and SV (shear waves with vertical motion)
  Reflection and refraction changing into P and S waves (page 61 Bolt), SH waves stay S-waves upon reflection/refraction.

- Rayleigh waves can appear in vertical and horizontal recorded accelerations.

- Love waves only appear in horizontal accelerations.

- Surface waves (Rayleigh and Love) spread out into a train of motion as they propagate (dispersion). Waves with longer wavelength travel quicker than waves with shorter wavelength, because longer waves penetrate deeper into the earth where the ground is stiffer and transmittal of waves is faster. Time of arrival of each wavelength in the surface train of waves can be used to back figure the dispersion relation and the elastic properties of rocks through which the waves traveled.
Field Arrangement Used in SASW Testing with a Common-Receivers Midpoint Geometry

Courtesy of K. H. Stokoe II
Schematic Representation of Rayleigh Waves of Different Wavelengths Sampling Material to Different Depths

Air

Rayleigh Wave Vertical Particle Motion

Layer 1

Layer 2

Layer 3

Depth

\( \sim \lambda_{R1} \)

\( \sim \lambda_{R2} \)

a. Material Profile

b. Shorter Wavelength, \( \lambda_{R1} \)

c. Longer Wavelength, \( \lambda_{R2} \)

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Courtesy of K. H. Stokoe II
Rayleigh Waves (Vertical Particle Motion)

- Variations in stiffness profile with depth
  - A: Shorter Wavelength
  - B: Longer Wavelength

- Surface Wave Velocity $V$ (ft/sec) vs. Wavelength of Surface Wave $\lambda$ (ft)
  - Variation in stiffness profile with depth
  - Graph showing $V$ vs. $\lambda$ for different stiffness profiles.
Sample seismic rays through the Earth are shown in the Figure. Begin at the focus of the earthquake F. The symbol c designates a wave reflected at the outer core’s surface; thus PcP is a P wave through the mantle reflected at the core; ScP is an S wave reflected as P.

The symbol i designates a wave reflected at the inner core’s boundary: PKiKP. The symbols K and I refer, respectively, to P waves that have traveled through the outer and inner core. The symbol SP designates an S wave through the mantle reflected at the outer surface as P. The ray marked PKJKP, which travels as S through the inner core, has not been observed. [From Bruce A. Bolt, *inside the Earth* (San Francisco: W. H. Freeman and Company, Copyright 1982).]
Time between arrivals of P- and S- waves

\[
\text{TIME} \quad \begin{align*}
\Delta t &= V_p t_i \\
d &= V_s \text{DISTANCE}
\end{align*}
\]
At any time $t_i$ the P-wave has gone a greater distance than the S-wave

$$d_p = V_p t_p \quad d_s = V_s t_s$$

At a given location away from the source, both waves traveled the same distance, i.e., $d_p = d_s = d$

Time for P-wave to arrive: $t_p = \frac{d}{V_p}$

Time for S-wave to arrive: $t_s = \frac{d}{V_s}$

Time between P- and S-wave to arrivals:

$$\Delta t = t_s - t_p = \frac{d}{V_s} - \frac{d}{V_p} = d \left( \frac{1}{V_s} - \frac{1}{V_p} \right)$$
Example: ROMANIA – 1977 earthquake

FOCUS

EPICENTER

V_p = 5.5 km/sec
V_s = 3.0 km/sec

A. How much time of warning did people have between P- and S-wave arrivals?

Focal Distance \( D_f = \sqrt{D_e^2 + F_D^2} = \sqrt{150^2 + 90^2} = 175 \text{ km} \)
Time for P-wave to reach Bucharest: 
\[ t_p = \frac{D_f}{V_p} = \frac{175\text{km}}{5\text{km/sec}} = 35\text{ sec} \]

Time for S-wave to reach Bucharest: 
\[ t_s = \frac{D_f}{V_s} = \frac{175\text{km}}{3\text{km/sec}} = 58\text{ sec} \]

Time between P- and S-wave to arrivals:
\[ \Delta t = t_s - t_p = 58\text{ sec} - 35\text{ sec} = 23\text{ sec} \]

**B. What is the shear modulus, G, for the rock?**

\[ \rho = 2250\text{kg/m}^3 \]
\[ V_s = 3000\text{m/s} \]

\[ G = \rho V_s^2 = 2250 \frac{\text{kg}}{\text{m}^3} \times (3000 \frac{\text{m}}{\text{s}})^2 = 20.25 \times 10^9 \frac{\text{kg} \cdot \text{m}}{\text{s}^2 \cdot \text{m}^2} = 20.25\text{GPa} \]
Alternate Forms for G, B, E and $\nu$

\[ G = \rho V_s^2 \]
\[ \nu = \text{Poisson’s Ratio} = \frac{0.5(V_p/V_s)^2 - 1}{(V_p/V_s)^2 - 1} \]
\[ B = \rho(V_p^2 - \frac{4}{3}V_s^2) \]
\[ E = 2\rho V_s^2 (1 + \nu) \]

**P-WAVE VELOCITIES**

<table>
<thead>
<tr>
<th>Material</th>
<th>$V_p$ (m/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>340</td>
</tr>
<tr>
<td>Soft soil</td>
<td>250-550</td>
</tr>
<tr>
<td>Stiff soil</td>
<td>450-600</td>
</tr>
<tr>
<td>Water</td>
<td>1500</td>
</tr>
<tr>
<td>Dense gravel</td>
<td>450-1200</td>
</tr>
<tr>
<td>Cemented gravel</td>
<td>1200-2000</td>
</tr>
<tr>
<td>Soft shale</td>
<td>900-2000</td>
</tr>
<tr>
<td>Hard shale</td>
<td>1800-3000</td>
</tr>
<tr>
<td>Sandstone</td>
<td>1500-3000</td>
</tr>
<tr>
<td>Limestone</td>
<td>2400-5500</td>
</tr>
<tr>
<td>Basalt</td>
<td>2400-4000</td>
</tr>
<tr>
<td>Granite</td>
<td>3000-6100</td>
</tr>
</tbody>
</table>
How to locate an earthquake

Method is based on time needed for travel of P-waves (S-waves). Currently, average travel time of seismic P-waves and S-waves is available for any specified distance, anywhere in the world (available in tables as a function of distance).

Today, 60 or more seismographic stations may be used to determine location of an earthquake (epicenter) using a computer.

If you use the common time-scale on seismic records, you can use P-wave arrivals alone to determine location.

If you do not have a common time scale, use S-P arrival times to determine travel distance using available tables and graphs (or use equations for P-wave and S-wave velocities with assumed elastic rock properties).
If you have 3 stations, get three distances and use triangulation to get the intersection between arcs.